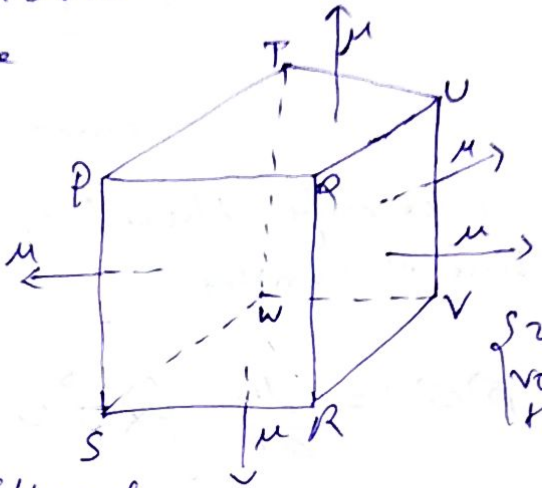
Relationship between Young's modulus and Bulk modulus:

Let us take a cube PQRSTUVW.
This cube is subjected to the mutually perpendicular tensile stresses of equal intensity.



$\{v = L^3$ volume of the cube

Next, let us take

l = length of the sides of the cube, Volume of the cube $= l^3$

dl = change in length $v = L^3$

$\mu \Rightarrow$ tensile stress acting on the faces

Y = Young's modulus of the material of the cube

σ = Poisson's ratio

Next, we consider the strain of one of the sides of the cube (PQ) under the action of three mutually perpendicular stresses.

i) strain ~~of PQ due~~ of PQ due to stresses on the faces PTWS and QUVR is tensile and equal to $\frac{\mu}{Y}$

ii) Strain of PQ due to stresses on the faces PTUQ and SWVR is compressive lateral strain and is equal to $-\sigma\frac{\mu}{Y}$

iii) Strain of PQ due to stresses on the faces PQRS and TUVW is also compressive lateral and is equal to $-\sigma \frac{\mu}{Y}$

Therefore, the ~~strain~~ total strain of PQ is obtained as

$$\frac{dl}{l} = \frac{\mu}{Y} - \sigma\frac{\mu}{Y} - \sigma\frac{\mu}{Y}$$

$$\text{or } \frac{dl}{l} = \frac{\mu}{Y}(1-2\sigma) \quad \text{—— (1)}$$

Next, the natural volume of the cube $v = l^3$

$dl \rightarrow$ change in length

$\Rightarrow dv \rightarrow$ change in volume.

$\because v = l^3$

$\therefore dv = 3l^2 dl$

$$\frac{dv}{v} = \frac{3l^2 dl}{l^3}$$

$$\text{or } \frac{dv}{v} = 3\frac{dl}{l} \quad \text{—— (2)}$$

From (1) and (2)

$$\frac{dv}{v} = \frac{3\mu}{Y}(1-2\sigma)$$

since bulk modulus is defined by $K = \frac{\mu}{\frac{dv}{v}}$

$$K = \frac{\mu}{\frac{3\mu}{Y}(1-2\sigma)} = \frac{Y}{3(1-2\sigma)}$$

$$\boxed{Y = 3K(1-2\sigma)}$$

or we can write $\boxed{\sigma = \frac{3K-Y}{6K}}$